**Damanhour University**
**Faculty of Science**
**Department of Mathematics**

| | |
|---|---|
| Course Code | Math 409 |
| Course Name | Differential Geometry |
| Program | Undergraduate |
| Section | Mathematics/physics |
| Level | Third |
| Total Marks | 70 |

| | |
|---|---|
| Term | Second |
| Date | 23/5/ 2015 |
| Start Time | 9:00 AM |
| End Time | 11:00 AM |
| Time Allowed | 2 hours |
| Dr. Name | Safaa Mosa |

# FINAL EXAM

**Answer the Following Questions**

## Question No. 1 (25 marks)

1- Define a $C^k$- coordinate patch and show that $f(x, y, z) = x^2 - y^2 - z$ gives a surface.

2- For any unit speed curve $\alpha(s) = \mathbb{X}(u^1(s), u^2(s))$, prove that $k_N = \sum_{i,j=1}^{2} L_{ij} \left(u^i\right)' \left(u^j\right)'$

3- Let $\alpha : I \to R^3$ be a regular parameterized curve (not necessarily by s). Prove that

a) $\frac{dt}{ds} = \frac{1}{|\dot{\alpha}|}$, $\frac{d^2t}{ds^2} = -\frac{\dot{\alpha} \cdot \ddot{\alpha}}{|\dot{\alpha}|^4}$

b) $k(t) = \frac{|\dot{\alpha} \times \ddot{\alpha}|}{|\dot{\alpha}|^3}$

## Question No. 2 (20 marks)

1- State and prove the Isoperimetric Inequality.

2- Define the geodesic for a surface $M$. Let $M$ be a surface of revolution generated by the unit speed curve $(r(t), z(t))$, show that every meridian of $M$ is a geodesic where $\mathbb{X}(t, \theta) = (r(t)\cos\theta, r(t)\sin\theta, z(t))$. (Note that all Christoffel symbols of $M$ are zero except $\Gamma_{22}^1 = -r(t)\dot{r}(t)$ & $\Gamma_{12}^2 = \Gamma_{21}^2 = \frac{\dot{r}(t)}{r(t)}$).

## Question No. 3 (25 marks)

1- For the simple surface $\mathbb{X}: U \to R$, prove that $\det(g_{ij}) = |\mathbb{X}_1 \times \mathbb{X}_2|$ and then show that a sphere of radius $a$ is of area $4\pi a^2$.

2- If $\tau \neq 0$ along $C$, show that more than one curves $C^*$ exists such that $C$ and $C^*$ are Bertrand curves if and only if $C$ is a circular helix.

3- Define the involute $\alpha^*$ of a curve $\alpha$ and find the involute for the helix $\alpha(t) = (a\cos t, a\sin t, bt)$.

**With my best wishes**

**Damanhour University**
**Faculty of Science**
**Department of Mathematics**

| | |
|---|---|
| Course Code | Math 458 |
| Course Name | Quantum Mechanics (2) |
| Program | Undergraduate |
| Section | Mathematic/Physics |
| Level | Fourth |
| Total Marks | 70 |

| | |
|---|---|
| Term | Second |
| Date | May 24th, 2015 |
| Start Time | 9:00 AM |
| End Time | 11:00 AM |
| Time Allowed | 2 hours |
| Dr. Name | Mohammed Hammad |

# FINAL EXAM

**Answer the following questions**

**Question No. 1** **(15 marks)**

1- Let $J$ is the total angular momentum operator of the coupling two angular momentum operators $J_1$ and $J_2$.

Prove $[J^2, J_1^2] = 0$.

2- Find the Clebsch- Gordan coefficents of $j_1 = 1$ and $j_2 = \frac{1}{2}$ (using recursion method).

Hint,

$$\sqrt{(j \mp m)(j \pm m + 1)}\langle j_1, j_2, m_1, m_2 | j, m \pm 1, j_1, j_2\rangle$$

$$= \sqrt{(j_1 \pm m_1)(j_1 \mp m_1 + 1)}\langle j_1, j_2, m_1 \mp 1, m_2 | j, m, j_1, j_2\rangle +$$

$$\sqrt{(j_2 \pm m_2)(j_2 \mp m_2 + 1)}\langle j_1, j_2, m_1, m_2 \mp 1 | j, m, j_1, j_2\rangle$$

**Question No. 2** **(20 marks)**

Prove that, the eigenfunctions of orbital angular momentum are given by

$$Y_{lm} = (-1)^m \sqrt{\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}} P_l^m(cos\theta) e^{im\varphi},$$

where

$$L_z = -i\hbar \frac{\partial}{\partial \varphi}$$

$$L^2 = -\hbar^2 \left( \frac{1}{sin\theta} \frac{\partial}{\partial \theta} \left( sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{sin^2\theta} \frac{\partial^2}{\partial \varphi^2} \right).$$

**Question No. 3** **(15 marks)**

Prove that, the recoupling coefficients (Racah coefficients) are given by

$$|(j_1 j_2) j_{12} j_3; jm\rangle = \sum_{j_{23}} U(j_1 j_2 j j_3; j_{12} j_{23}) |j_1 (j_2 j_3) j_{23}; jm\rangle$$

**Please turn over**

**Question No. 4** **(20 marks)**

Show that the Clebsch-Gordon coefficients of coupling the orbital angular momentum $l$ and the spin angular momentum $=\frac{1}{2}$ are given from the following table

| | $\left\|m_1 = m-\frac{1}{2}, m_2 = \frac{1}{2}\right\rangle$ | $\left\|m_1 = m+\frac{1}{2}, m_2 = -\frac{1}{2}\right\rangle$ |
|---|---|---|
| $\left\|l+\frac{1}{2}, m\right\rangle$ | $\sqrt{\frac{l+\frac{1}{2}+m}{2l+1}}$ | $\sqrt{\frac{l+\frac{1}{2}-m}{2l+1}}$ |
| $\left\|l-\frac{1}{2}, m\right\rangle$ | $\sqrt{\frac{l+\frac{1}{2}-m}{2l+1}}$ | $-\sqrt{\frac{l+\frac{1}{2}+m}{2l+1}}$ |

**With my best wishes**

Damanhour University
Faculty of Sciences
Mathematics Department

Second Term Exam For The School Year 2014-2015

Level: One
Subject: Computer Programming (Math 150)
Time: 2:00 hours

Department: Mathematics

---

Answer the following questions :-

Q1: (a) Write a Fortran program to compute the factorial function n!, handle the illegal values (n<0).

(b) Write a Fortran program that determines the real roots of a quadratic equation $ax^2 + bx + c = 0$. When the program runs it asks the user to enter the values of the constants a, b, and c. To calculate the roots of the equation the program calculates the discriminate D given by: $D = b^2 - 4ac$.

If $D > 0$ the program displays a message: "The equation has two roots," and the roots are displayed in the next line .

If $D = 0$ the program displays a message: "The equation has one root," and the root is displayed in the next line .

If $D < 0$ the program displays a message: "The equation has no real roots ".

(c) Write a Fortran program to compute the square roots of the integers 1 to 10.

(d) What will be the results of executing the following Fortran program?

```
PROGRAM B
DO i = 1, 4
   i = 2
END DO
END PROGRAM B
```

Q2:

(a) Write down the following statements in your answer sheet and fill in the empty spaces:

```
(i)
>> A = [5 9 2 4 11 6 7 11 0 1];
>> C = max(A)
C =
...
>> A = [5 9 2 4];
>> sum(A)
ans =
...
```

```
(ii)
>> A= [5 9 2 4];
>> sort (A)
ans =
...
>> v =[7 4 2];
>> A = diag(v)
A=
...
```

```
(iii)
>> ceil(2/3)
...
>> fix(4/3)
...
>> round(5/4)
....
```

(b) Write a Matlab program to read 5 numbers from keyboard in the command window and compute them average.

(c) Write a Matlab code to plot the function:

$$f(x)=\begin{cases}0.6x^3-5x^2+9x+2 & -4<x<4\\ 3x^2+5 & 5<x<7\end{cases}$$

With Best Wishes

Course : M 102
Term: 2nd term 2014-2015
Time allowed: 2hours
Date: 27 /5/2015
Course : Calculus (2) & Algebra

Damanhur University
Faculty of Science
Department of Mathematics
Year : First year (علوم طبيعية)

## Calculus (2) :

Answer The Following Questions :

### First Question:

a) Using integration by parts to prove the following recurrence formula $\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1} x\, \sin x + \frac{n-1}{n}\int \cos^{n-2} x\, dx$ .

Hence , evaluate $\int \cos^5 x\, dx$ .

b) Compute the integrals

1) $\int \sqrt{x^2 - 25}\, dx$

2) $\int \frac{dx}{(x^2 + 2x + 10)^2}$

c) Calculate the length of the arc of the curve $y = x^2$ between $x = 0$ and $x = 2$.

### Second Question:

a) Calculate the integrals

1) $\int \frac{dx}{2 + \cos x}$

2) $\int_0^{\frac{\pi}{2}} \sin^2 x\, \cos^4 x\, dx$

b) Find the area bounded by the two curves $y = x^2 + 3x + 5$ and $y = -x^2 + 5x + 9$ for $x$ between $-1$ and $4$ .

[3] a- If one roots of the equation $f(x) = 2x^4 + x^3 + 5x^2 + 4x - 12 = 0$

Is pure imaginary. Find all roots of this equation

b- Find the modulus and argument of the complex number $(1+i)^{1+i}$

c- Find the partial fraction for $\frac{x^2}{(x^4-81)(x-1)^2}$ ,

d-By using the mathematical induction rule prove that

$3^{2n+1} + 2^{n+2}$ is divisible of 7 $\forall n \in Z^+$.

**Department: Mathematics** **Section: Physics/chemistry**

**Year: fourth** **Term: second, 2014/2015**

**Course Code: Math 350** **Total points: 70**

**Time: 2 hours**

**Course title: Math. Programming and Numerical Methods**

**Dr. Name: Faisal Ezz Eldeen** **Date: 31/5/2015**

**Question 1** 1- Find the first two iterations of the **Jacobi method** using $x^{(0)} = y^{(0)} = z^{(0)} = 0$.

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

2- Write a computer program for the **Jacobi** method.

**Question 2**

1- Use Trapezoidal rule to approximate

$$\int_1^2 x \ln x \, dx, \qquad n = 4.$$

2- Use the following data to approximate $f'(0.7)$ and $f'(0.6)$

| $x$ | 0.5 | 0.6 | 0.7 |
|---|---|---|---|
| $f(x)$ | 0.4794 | 0.5646 | 0.6442 |

**Question 3**

1- Use Euler's method to approximate the solutions for the problem

$$y' = y - t^2 + 1, \ for \ 0 \le t \le 2, \ with \ y(0) = 0.5, n = 10.$$

2- Write a computer program for Euler's method.

**Question 4**

1- Use Lagrange interpolating polynomial of degree 2 to approximate $f(8.4)$ *if* $f(8.3) = 17.56$, $f(8.6) = 18.51$, $f(8.7) = 18.82$.

**ملاحظة هامة**

**تكملة الاختبار خلف الورقة**

2- Use Newton's method to find $p_2$ for $x^3 + 3x^2 - 1 = 0$, $p_0 = -3$.

3- Write a computer program for the Newton's method.

---

**GOOD LUCK**

Course : M 102
Term: 2nd term 2014-2015
Time allowed: 2hours
Date: 27 /5/2015
Course : Calculus (2) & Algebra

Damanhur University
Faculty of Science
Department of Mathematics
Year : First year (علوم طبيعية)

## Calculus (2) :

Answer The Following Questions :

### First Question:

a) Using integration by parts to prove the following recurrence formula $\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1} x\, \sin x + \frac{n-1}{n}\int \cos^{n-2} x\, dx$ .

Hence , evaluate $\int \cos^5 x\, dx$ .

b) Compute the integrals

1) $\int \sqrt{x^2 - 25}\, dx$

2) $\int \frac{dx}{(x^2 + 2x + 10)^2}$

c) Calculate the length of the arc of the curve $y = x^2$ between $x = 0$ and $x = 2$.

### Second Question:

a) Calculate the integrals

1) $\int \frac{dx}{2 + \cos x}$

2) $\int_0^{\frac{\pi}{2}} \sin^2 x\, \cos^4 x\, dx$

b) Find the area bounded by the two curves $y = x^2 + 3x + 5$ and $y = -x^2 + 5x + 9$ for $x$ between $-1$ and $4$ .

[3] a- If one roots of the equation $f(x) = 2x^4 + x^3 + 5x^2 + 4x - 12 = 0$

Is pure imaginary. Find all roots of this equation

b- Find the modulus and argument of the complex number $(1+i)^{1+i}$

c- Find the partial fraction for $\frac{x^2}{(x^4-81)(x-1)^2}$ ,

d-By using the mathematical induction rule prove that

$3^{2n+1} + 2^{n+2}$ is divisible of 7 $\forall\, n \in Z^+$.

**Department:** Mathematics
**Year:** Second
**Section:** Mathematical/physics
**Course Code:** Math 202
**Course Title**: Mathematical Analysis ( 2)
**Term:** Second, 2012/2013
**Date:** / 5 /2013
**Time Allowed:** 2 hours
**Total points:** 70 point
**Dr. Name(s): Dr.** Fatma Gaafar

جامعة دمنهور
كلية العلوم

استعن بالله أولا

**Solve the following questions**

(1) **a-** Define the convergence of a sequence $\{a_n\}$ --

and show that If $a_n = \frac{n!}{n^n}$, then $\lim_{n\to\infty} a_n = 0$.

**b-** Prove that every convergent sequence is bounded -- give an example to prove that the converse is not true.

**c-** Discuss the monotonictelly, boundeness, and convergent for the sequence $\{a_n\}$ where $a_1 = \sqrt{3}$ , $a_{n+1} = \sqrt{3\,a_n}$ for $n = 1, 2, 3, 4,$ ... (state the theorem you are used).

---

(2) **a-** Find the Fourier series for the function $f(x) = x^2$ , $-\pi < x < \pi$ and sketch the periodic function.

Hence prove that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

**b-** Define the absolute and conditionally convergence of the alternative series.

**Please turn over**

c- Test for absolute and conditionally convergence for the series (state the theorem you are used)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\log n}{n^2} \quad , \quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{n!}$$

---

(3) a- What we mean by improper integrals -- Examine the convergence of the improper integrals

$$\int_0^3 \frac{1}{(x-1)^3}\, dx \quad \forall p \in R \quad , \quad \int_0^{\infty} \frac{dx}{x^p} \, , p > 0 \quad , \quad \int_0^1 \frac{dx}{\sqrt{x - x^2}}$$

---

(4) a- Prove that $\lim_{n \to \infty} a_n = 0$ is a necessary condition for convergence of an infinite series $\sum_{n=1}^{\infty} a_n$ but is not a sufficient condition for convergence.

b- Test for convergence (State the tests you are use in each case)

(i) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ find its sum if possible.

(ii) $\sum_{n=1}^{\infty} n\, e^{-n^2}$ (iii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-5)^n}{n\, 5^n} \quad \forall x \in R$

---

GOOD LUCK مع أطيب التمنيات بالنجاح والتوفيق

جامعة دمنهور

كلية العلوم

**Department: Mathematics** | **Section: Math/Physics**
**Year: second** | **Term: second, 2014/2015**
**Course Code: Math 306** | **Total points: 70**
**Course title: Numerical Analysis 1** | **Time Allowed : 2hours**
**Dr. Name: Faisal Ezz Eldeen** | **Date: 26/5/2015**

**Question 1** 1- Find the first two iterations of the **Jacobi method** using

$$x^{(0)} = y^{(0)} = z^{(0)} = 0$$

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

$$6x + 3y + 12z = 35$$

2- Use Trapezoidal rule to approximate

$$\int_0^{0.4} \frac{1}{1 + x^2}\, dx , \qquad n = 4 .$$

**Question 2**

1- Use the Trapezoidal rule error formula to determine the values of ***h*** and ***n*** required to approximate

$$\int_0^2 \frac{1}{1 + x}\, dx ,$$

to within $\mathbf{10^{-5}}$.

2- Use Lagrange interpolating polynomial of degree 2 to approximate $f(8.4)$ *if* $f(8.3) = 17.56$, $f(8.6) = 18.51$, $f(8.7) = 18.82$.

**Question 3**

1-Find a bound for the number of Bisection method iterations needed to achieve an approximation with accuracy $10^{-4}$ to the solution of $x^3 - x - 1 = 0$ lying in the interval $[1, 2]$

2-Show that the Fixed point iteration method converges linearly.

ملاحظة هامة

تكملة الاختبار خلف الورقة

**Question 4**

- Given the data

| $x$ | 0.5 | 0.6 | 0.7 |
|---|---|---|---|
| $f(x)$ | 0.4794 | 0.5646 | 0.6442 |

1- Use these data to approximate $f'(0.5)$ and $f'(0.7)$

2-These data were taken from $f(x) = \sin x$. Find the error bounds using the error formulas.

---

**GOOD LUCK**

| | | | | | | |
|---|---|---|---|---|---|---|
| | رقم ورمز المقرر | المنطق الرياضى | المقرر | الرياضيات | القسم | |
| ساعتين | زمن الامتحان | رياضيات\ فيزياء | الشعبة | الثانية | الفرقة | |
| ١٠٠ | درجة الاختبار | الثانى | الفصل الدراسى | ٢٠١٤ـ ٢٠١٥ | العام الدراسى | |

**Answer the following questions**

1- Construct the truth table of $(p \wedge q) \rightarrow p$

2- Prove that the two propositions $\sim(p \leftrightarrow q)$, $(p \wedge \sim q) \vee (\sim p \wedge q)$ are logically equivalent

3- Using the laws of algebra of propositions , show that $(p \rightarrow q) \wedge (r \rightarrow q) \equiv (p \wedge r) \rightarrow q$

4- Rewrite the expression $p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$ in a form which does not involve conditional connective as well as the bi conditional connective

5- Write down the inverse and converse of the proposition

$$\exists_x \forall_y [P(x,y) \wedge \forall_x Q(x,y)] \vee \sim Q(x,y)$$

6- By using Boolean algebra, Simplify the Boolean expression $(X + Y + Z)(Y + Z)Z$

7- By using Karnaugh Map, simplify the Boolean expression

$$\overline{A}\,\overline{B}\,C\,D + \overline{A}\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D}$$

8- Draw the logic circuit of HA. By using HA draw the logic circuit of FA

9- Draw a series of parallel 4 bit binary adder to add the numbers 1100 and 1111

Damanhour University
Faculty of Sciences
Mathematics Department

Second Term Exam For The School Year 2014-2015

Level: Two
Group: (Math/ Phy)
Subject : Introduction to Probability Theory and Mathematical Statistics (Stat. 102)
Time: 2:00 hours

---

Answer the following questions :-

Q1:

(a) Suppose that you daily ride a commuter train from your home into faculty (in Damanhour). The station you leave from has trains leaving for Damanhour at 7:00 a.m., 7:13 a.m., 7:20 a.m., 7:25 a.m., 7:32 a.m., 7:45 a.m., and 7:55 a.m. It is your practice to take the first train that leaves after your arrival at the station. Due to the vagaries of your rising time and traffic you encounter deriving to the station, you are equally likely to arrive at the station at any instant between 7:15 a.m. and 7:45 a.m. On a particular day, what is the probability you have to wait less than 5 minutes at the station? Less than 10 minutes? Suppose the 7:25 a.m. and 7:45 a.m. trains are express. What is the probability that you catch an express on a given day?

(b) Prove that if $A$, $B$ are independent so $A^c and\ B^c$.

(c) The Damanhour University football team plays 11 different teams on successive Saturdays. If we assume that their performances from one Saturday to the next are independent with probability 0.6 to win. What is the probability that they win all their games?

Q2:

(a) Many manufacturers have quality control programs that include inspection of incoming materials for defects. Suppose that a computer manufacturer receives computer boards in lots of 5. Two boards are selected from each lot for inspection. We can represent possible outcomes by pairs. For example (1,2) represents selection of boards 1 and 2 for inspection.

(i) List the 10 different possible outcomes.

(ii) Suppose that boards 1 and 2 are the only defective boards in a lot of 5. Two boards are to be chosen at random. Define $X$ to be the number of defective boards observed among those inspected. Find the probability distribution of $X$.

(b) Let X be a continuous random variable with:

$$f_X(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & otherwise. \end{cases}$$

Compute: (i) $E[X+10]$ (ii) $\sigma_X$.

(c) Suppose that the reaction time Y to a certain stimulus has normal distribution with mean μ and standard deviation σ. What is the probability that a particular reaction time will fall within 1 standard deviation of the mean ( $P(\mu - \sigma < Y < \mu + \sigma)$)?

Q3:

(a) Mohamed takes a written driver's license test repeatedly until he passes it. Assume the probability that he passes it any given time is 0.1 and that the tests are independent. Compute the probability that:

(i) it takes him more than 4 attempts

(ii) it takes him at most 10 attempts .

(b) Assume that the distance $X$ that a particular athlete will be able to put a shot (on his first try) is a normal random variable with parameters $\mu = 50$ ft., $\sigma = 5$ ft. Compute the probability that he tosses it no less than 55 ft., and the probability that his toss travels between 50 ft., and 60 ft.

---

With Best Wishes

ملحوظة: يسمح للطالب إحضار الجداول الإحصائية